

# ***Nuclear response functions for the N-N\*(1440) transition***

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# Introduction

- Second resonance region:

$N^*(1440)P_{11}$  (**Roper**),  $N^*(1520)D_{13}$ ,  $N^*(1535)S_{11}$

## The Roper Resonance

$$I(J^P) = 1/2(1/2^+)$$

$$\Gamma = 250 - 450 \text{ MeV} \leftarrow \text{PDG}$$

Decay modes:	$N\pi$	60-70 %
	$N\pi\pi$	30-40 %
	$\Delta\pi$	20-30 %
	$N\rho$	$\leq 8$ %
	$N(\pi\pi)_{s-wave}^{I=0}$	5-10 %

- Plays a **relevant role** in several hadronic reactions:

$\alpha p \rightarrow \alpha X$ ,  $\pi N \rightarrow \pi\pi N$ ,  $NN \rightarrow NN\pi\pi$ ,  $\gamma N \rightarrow \pi\pi N$

Different descriptions :

- Radial excitation of the nucleon
  - Relativistic corrections
  - Meson clouds
  - Configuration mixing
- Chiral soliton
- Hybrid state with large gluonic component
- $N\text{-}\sigma$  molecule



Different behaviors of the **helicity amplitudes**

# PC response functions

- For the  $(e, e')$  reaction:

$$\frac{d\sigma}{d\Omega' dE'} = \sigma_M \frac{1}{2EE' \cos^2 \theta_e / 2} L_{\mu\nu} W^{\mu\nu} = \sigma_M (v_L R^L + v_T R^T)$$

$\sigma_M$  ← Mott c.s.,  $v_{L(T)}$  ← kinematical factors

- Within the Relativistic Fermi Gas model

$$R^{L(T)}(|\vec{q}|, \omega) = \int_{\mu_{min}^*}^{\mu_{max}^*} d\mu^* G(\mu^*) R_0^{L(T)}(|\vec{q}|, \omega, \mu^*)$$

$G(\mu^* = W/m_N) \leftarrow N^*(1440)$  spectral function

$R_0^{L(T)} \sim U^{L(T)} = f(\text{kinematical variables}, w_{1,2})$

- Single-nucleon tensor:

$$f^{\mu\nu} = -\textcolor{red}{w}_1(\tau, \mu^*) \left( g^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\tau} \right) + \textcolor{red}{w}_2(\tau, \mu^*) V^\mu V^\nu$$

- Dimensionless variables

$$\eta = (\epsilon, \vec{\eta}) = \left( \sqrt{1 + \frac{\vec{p}^2}{m_N^2}}, \frac{\vec{p}}{m_N} \right) \leftarrow \text{nucleon}$$

$$\kappa = (\lambda, \vec{\kappa}) = \left( \frac{\omega}{2m_N}, \frac{\vec{q}}{2m_N} \right) \leftarrow \text{virtual photon}$$

$$\tau = \vec{\kappa}^2 - \lambda^2 = -\frac{q^2}{4m_N^2} \leftarrow \text{4-momentum transfer}$$

$$V_\mu = \eta_\mu + \kappa_\mu \rho$$

$$\rho = 1 + \frac{1}{4\tau}(\mu^{*2} - 1) \leftarrow \text{inelasticity}$$

- All info about the  $N - N^*$  transition is in  $\textcolor{red}{w}_{1,2}$

# Hadronic tensor and form factors

- The  $N - N^*(1440)$  tensor:

$$\begin{aligned} f^{\mu\nu} &= \frac{1}{2}\mu^* \text{Tr} \left[ \frac{(\not{p} + m_N)}{2m_N} \left( \gamma_0 J^{\dagger\mu} \gamma_0 \right) \frac{(\not{p}' + W)}{2W} J^\nu \right] \\ &= -w_1 \left( g^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\tau} \right) + w_2 V^\mu V^\nu \end{aligned}$$

- The transition current:

$$J^\alpha = \bar{u}_{N^*}(p') \left[ F_1(q^2) (\not{q} q^\alpha - q^2 \gamma^\alpha) + i F_2(q^2) \sigma^{\alpha\beta} q_\beta \right] u(p)$$



# **Hadronic tensor and form factors**

$$w_1 = \left[ \tau + \frac{(\mu^* - 1)^2}{4} \right] G_M^2$$
$$w_2 = \frac{1}{1 + \frac{4\tau}{(\mu^* + 1)^2}} \left[ G_E^2 + \frac{4\tau}{(\mu^* + 1)^2} G_M^2 \right]$$

$G_{E,M}$  are defined analogously to the  
**Sachs form factors** of the nucleon

$$G_E = 4m_N^2 \tau \left[ F_1 - \frac{F_2}{m_N(\mu^* + 1)} \right]$$
$$G_M = 4m_N^2 \tau F_1 + m_N(\mu^* + 1) F_2$$

# ***Helicity amplitudes***

$$A_{1/2}^{p(n)}(Q^2) = \sqrt{\frac{2\pi\alpha}{k_R}} \langle N^* \downarrow | \epsilon_\mu^{(+)} J^{p(n)\mu} | N \uparrow \rangle$$
$$S_{1/2}^{p(n)}(Q^2) = \sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{Q^2} \langle N^* \uparrow | \epsilon_\mu^{(0)} J^{p(n)\mu} | N \uparrow \rangle,$$

↓

$$A_{1/2}^{p(n)} \sim G_M^{p(n)} \quad S_{1/2}^{p(n)} \sim G_E^{p(n)}$$

# ***Helicity amplitudes***

Theoretical calculations:

- Non-relativistic quark model
- Hybrid model

Li, Burkert, Li, Z.Phys C '80

- Light front relativistic quark model

Cardarelli et al., PLB 397 '97

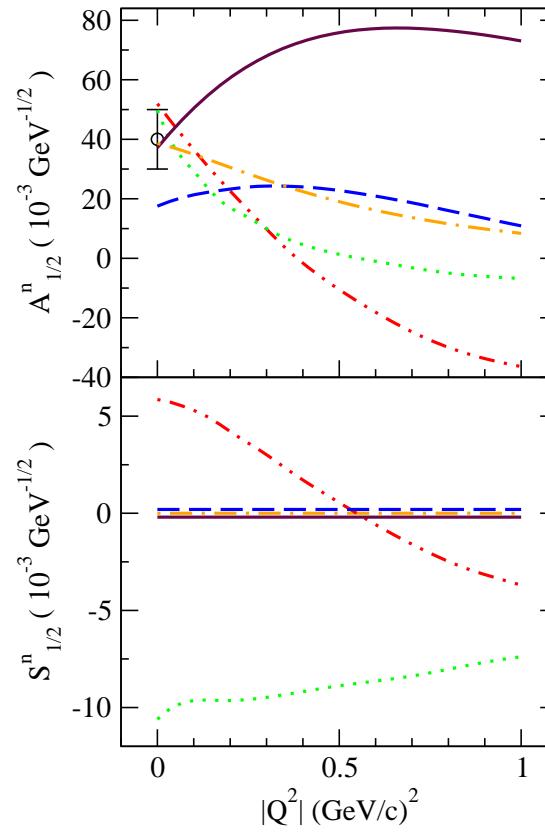
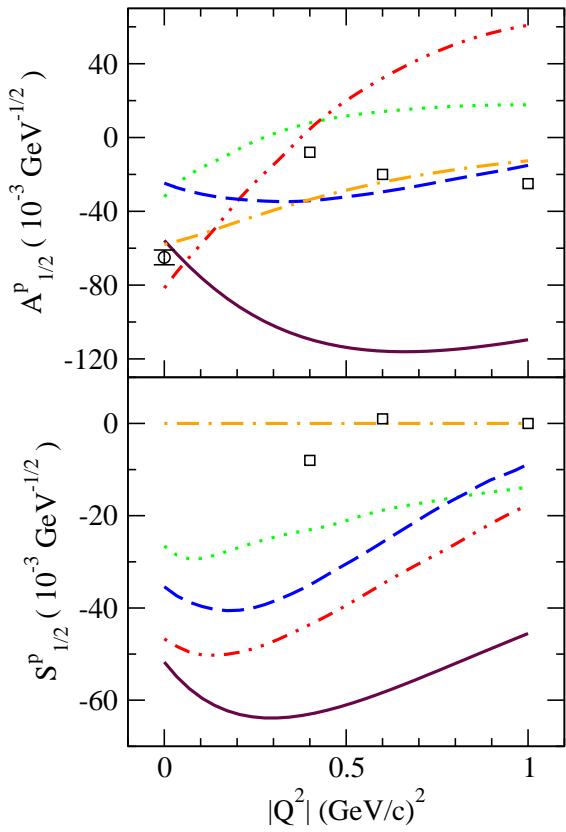
- Chiral chromodielectric model

Alberto et al. PLB 523 '01

- Extended vector-meson dominance model

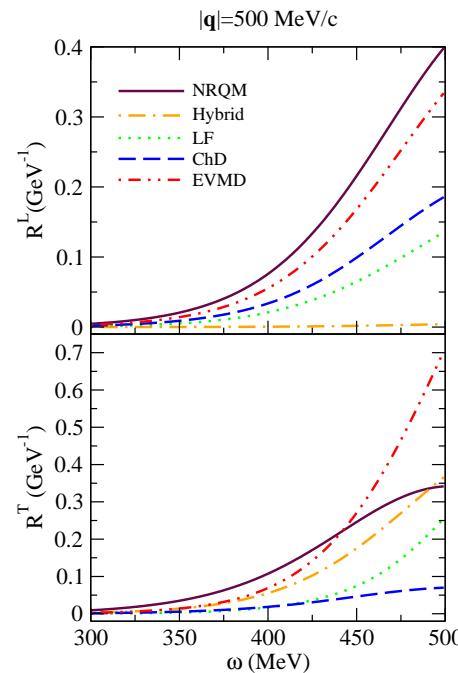
Cano, Gonzalez, PLB 431 '98

# *Helicity amplitudes*



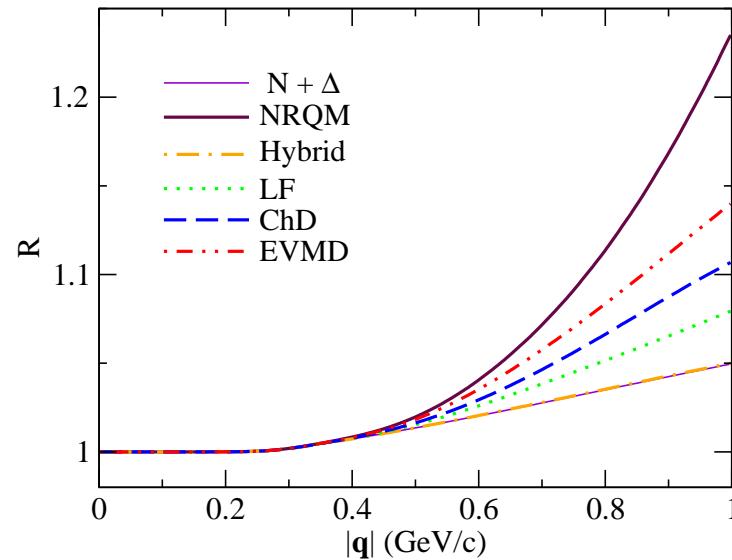
# Results

- $^{12}C \Leftrightarrow k_F = 225 \text{ MeV}/c, Z = N = 6$
- Negligible contribution to  $R^T$  compared to the  $\Delta$
- Sizable and model dependent contribution to  $R^L$



# **Resonance contribution to the Coulomb sum rule**

$$\mathcal{R}(|\vec{q}|) = \frac{\int_0^{|\vec{q}|} d\omega [R_{QE}^L(|\vec{q}|, \omega) + R_{\Delta, N^*}^L(|\vec{q}|, \omega)]}{\int_0^{|\vec{q}|} d\omega R_{QE}^L(|\vec{q}|, \omega)}$$



$\Delta \sim 5 \%$ ,  $N^*(1440) \sim 0 - 20 \%$

# PV response functions

- PV effects can be accessed in **polarized electron scattering** exp.

$$\mathcal{A} = \left( \frac{d\sigma}{d\Omega' dE'} \right)^{(PV)} \Bigg/ \left( \frac{d\sigma}{d\Omega' dE'} \right)^{(PC)} \quad \text{← asymmetry}$$

$$\left( \frac{d\sigma}{d\Omega' dE'} \right)^{(PV)} \equiv \frac{1}{2} \left( \frac{d\sigma^+}{d\Omega' dE'} - \frac{d\sigma^-}{d\Omega' dE'} \right) = f(\tilde{R}^L, \tilde{R}^T, \tilde{R}^{T'})$$

$$\tilde{R}^{L,T,T'}(|\vec{q}|, \omega) = \int_{\mu_{min}^*}^{\mu_{max}^*} d\mu^* G(\mu^*) \tilde{R}_0^{L,T,T'}(|\vec{q}|, \omega, \mu^*)$$

$$\tilde{R}^{L,T,T'} \sim \tilde{U}^{L,T,T'} = f(\text{kin. variables}, \tilde{w}_{1,2,3})$$

- The dynamical information is carried by  $\tilde{w}_{1,2,3}$

# Hadronic tensor and form factors

- PV  $\tilde{f}^{\mu\nu}$  ← interference between EM and NC

$$\begin{aligned}\tilde{f}^{\mu\nu} &= \frac{1}{2}\mu^* \text{Tr} \left[ \frac{(\not{p} + m_N)}{2m_N} \left( \gamma_0 J_{em}^{\dagger\mu} \gamma_0 \right) \frac{(\not{p}' + W)}{2W} J_{nc}^{\nu} \right] \\ &= -\tilde{w}_1 \left( g^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\tau} \right) + \tilde{w}_2 V^\mu V^\nu - i \tilde{w}_3 \epsilon_{\mu\nu\alpha\beta} \kappa^\alpha V^\beta\end{aligned}$$

- $J_{nc}^\alpha = J_V^\alpha + J_A^\alpha$  with

$$J_V^\alpha = \bar{u}_{N^*}(p') \left[ \tilde{F}_1 \left( \not{q} q^\alpha - q^2 \gamma^\alpha \right) + i \tilde{F}_2 \sigma^{\alpha\beta} q_\beta \right] u(p)$$

$$J_A^\alpha = \bar{u}_{N^*}(p') \left[ \tilde{G}_A \gamma^\alpha \gamma_5 + \tilde{G}_P q^\alpha \gamma_5 \right] u(p)$$



# Hadronic tensor and form factors

$$\tilde{w}_1 = \left[ \tau + \frac{(\mu^* - 1)^2}{4} \right] G_M \tilde{G}_M$$

$$\tilde{w}_2 = \frac{1}{1 + \frac{4\tau}{(\mu^* + 1)^2}} \left[ G_E \tilde{G}_E + \frac{4\tau}{(\mu^* + 1)^2} G_M \tilde{G}_M \right]$$

$$\tilde{w}_3 = G_M \tilde{G}_A$$

with

$$\tilde{G}_E = 4m_N^2 \tau \left[ \tilde{F}_1 - \frac{\tilde{F}_2}{m_N(\mu^* + 1)} \right]$$

$$\tilde{G}_M = 4m_N^2 \tau \tilde{F}_1 + m_N(\mu^* + 1) \tilde{F}_2$$

as in the EM case

# Form factors

$$2\tilde{G}_{E,M}^{p(n)} = (1 - 4 \sin^2 \theta_W) G_{E,M}^{p(n)} - G_{E,M}^{n(p)}$$

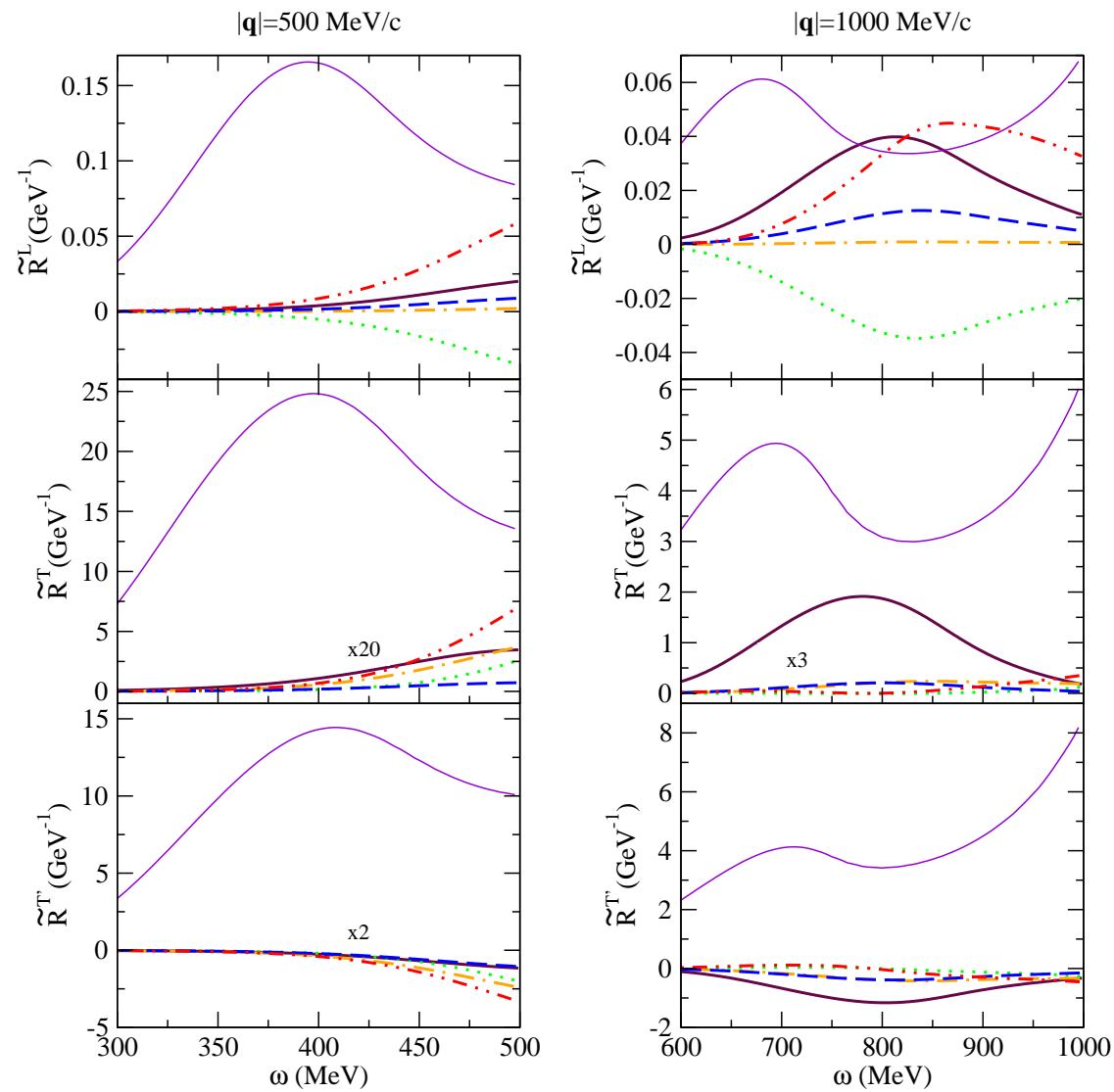
$$2\tilde{G}_A^{p(n)} = G_A^{p(n)} - G_A^{n(p)} = \pm G_A^V$$

- Vector ff are related to the EM ones
- There is no experimental info on  $G_A^V$ :

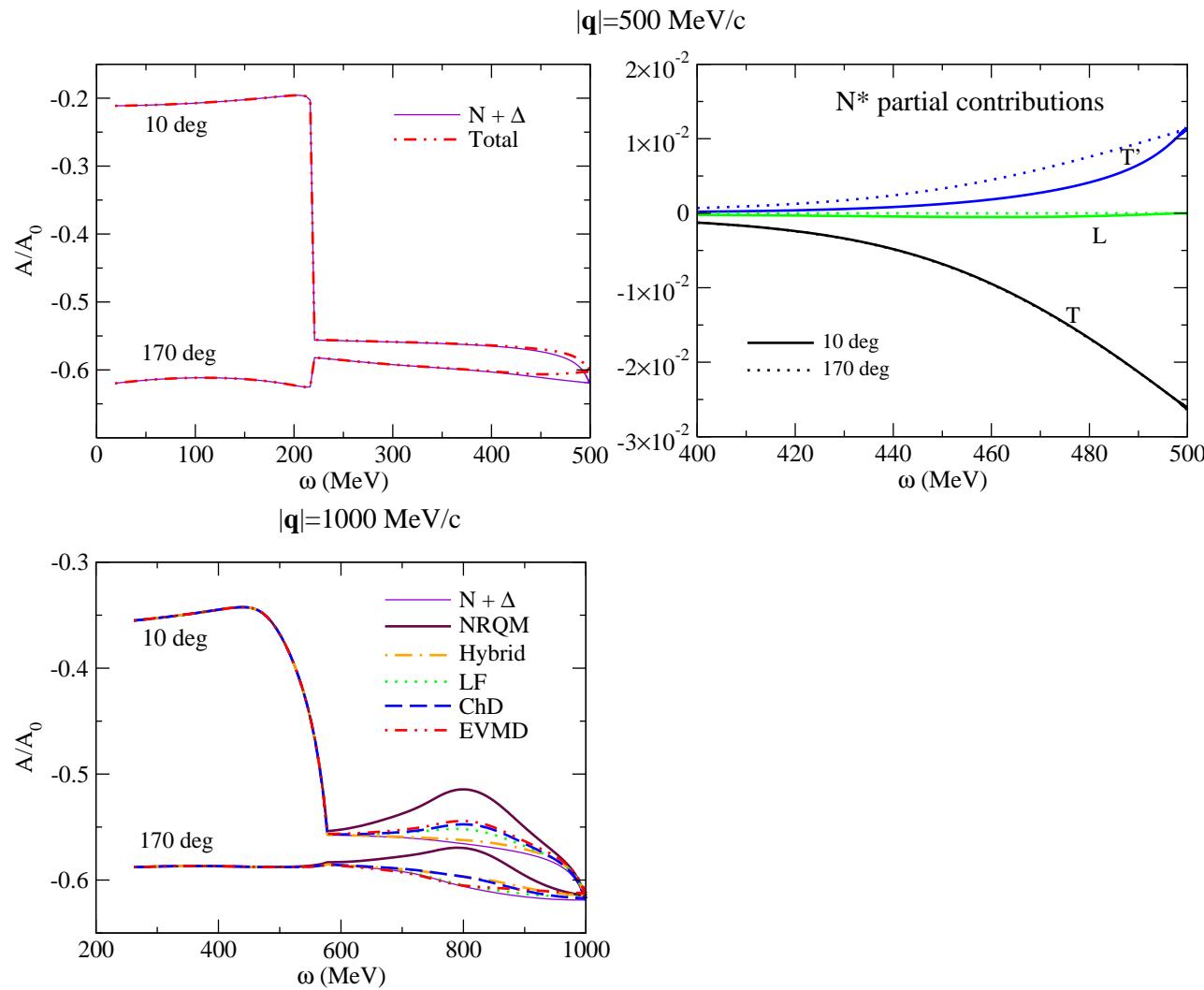
- $G_A^V(Q^2) = 2f_\pi \frac{\tilde{f}}{m_\pi} \left(1 + \frac{|Q^2|}{M_A^2}\right)^{-2}$

- Dipole  $q^2$  dependence with  $M_A = 1 \text{ GeV} \sim$  nucleon
- $\tilde{f} \leftarrow N^* \rightarrow N \pi$  coupling

# Results



# Asymmetry



# Conclusions

- The response functions (PC, PV) show sensitivity to the various  $N^*(1440)$  models.
- At low momentum transfer  $\sim 500 \text{ MeV}/c$ , heavier baryon resonances should not be too disruptive.
- Although the Roper  $R^L$  is large compared to the  $\Delta$  contribution close to the light cone, an experimental study might be hard.
- The contribution of the  $N^*(1440)$  to the Coulomb sum rule can be significant (up to 20 %).
- A complete study including  $N^*(1520)D_{13}$  and  $N^*(1535)S_{11}$  is needed.